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Quantum operators and their integral inequalities

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Abstract: **In the present paper authors derived certain new results of the bounded and continuous functions in relationship with quantum calculus operators in particular q- Reimann fractional operator. Some special cases of the main results also pointed out in the lost section of paper.**

Keywords: **Quantum calculus operators *and q-Gauss Hypergeometric function.***

1. INTRODUCTION AND PRELIMINARIES

The purpose of this article is to increase the accessibility of different dimensions of q-fractional calculus and generalization of basic hypergeometric functions to the real world problems of engineering, science and economics. Present paper reveals a brief history, definition and applications of basic hypergeometric functions and their generalizations in light of different mathematical disciplines of calculus, like quantum calculus.

Riemann-Liouville q-fractional operator

Agarwal [8], introduced the q-analogue of the Reimann-Liouville fractional integral operator as follows.

$$\mathrm{I}_{q,a}^{\alpha}\mathrm{f}(x) = \frac{1}{\Gamma_q(\alpha)}\int_a^x (x-\mathrm{q}t)^{\alpha-1}\mathrm{f}(t)\,\mathrm{d}_q(t)$$

Where α is an arbitrary order of integration such that Re $(\alpha)>0$.

Jackson [11], Al.Salam [9] and Agarwal [8] defined basic integration as

$$\int_0^x \mathrm{f}(t)\mathrm{d}_q(t) = x(1\text{-}q)\sum_{k=0}^{\infty} q^k \mathrm{f}(xq^k)$$

From above two equations, we get

$$\mathrm{I}_q^{\mu}\mathrm{f}\,(x) = \frac{x^{\mu}(1-q)}{\Gamma_q(\mu)}\sum_{k=0}^{\infty} q^k\left(1-q^k\right)_{\mu-1}\mathrm{f}(xq^k)$$

Saigo's q- Integral Operator

A basic analogue of Saigo's fractional integral operator [29] is defined as

$$\mathrm{I}_q^{\alpha,\beta,\eta}\mathrm{f}(x) = \frac{x^{-\beta-1}}{\Gamma_q(\alpha)}\int_0^x\left(\frac{tq}{x};q\right)_{\alpha-1}\times$$

$$\sum_{m=0}^{\infty}\frac{(q^{\alpha+\beta},q)_m(q^{-\eta},q)_m}{(q^{\alpha},q)_m(q;q)_m}q^{(\eta-\beta)m}(-1)^m(q)^{-\binom{m}{2}}\left(\frac{t}{x}-1\right)_m \mathrm{f}(t)\,\mathrm{d}_q(t).$$

And

$$f(x)=\frac{q^{-\binom{\alpha}{2}-\beta}}{\Gamma_q(\alpha)}\int_0^x\left(\frac{x}{t};q\right)_{\alpha-1}t^{-\beta-1}\times$$

$$\sum_{m=0}^{\infty}\frac{(q^{\alpha+\beta},q)_m(q^{-\eta},q)_m}{(q^{\alpha},q)_m(q;q)_m}q^{(\eta-\beta)m}(-1)^m(q)^{-\binom{m}{2}}\left(\frac{x}{qt}-1\right)_m f\left(tq^{1-\alpha}\right)d_q(t).$$

2. MAIN RESULTS

In this section, we obtain certain q- integral inequalities, related to the integrable functions, whose bounds are also integrable functions, involving q-Saigo's fractional hypergeometric operators. The results are given in the form of the following theorems.

Theorem1: *Let q_1 , q_2 $\in(0,t)$, f, g and h be three continuous and integrable functions defined on [0,∞) and u: [0,∞)→ $[0,\infty)$ such that $f(t)\le g(t)\le h(t)$ ∀ t ∈ [0,∞).*

$$I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)g(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)g(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)h(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)h(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)g(t)h(t)\}$$

$$\ge I_{q_2}^{\gamma,\delta,\xi}\{u(t)g(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)g(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)g(t)h(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)g(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)h(t)\}.$$

Proof: Let *f, g and h be three continuous and integrable functions on [0,∞) , then ∀ $\tau,\rho\ge 0$. we have*

$$(f(\tau)-f(\rho))(g(\tau)-g(\rho))(h(\tau)+h(\rho))\ge 0$$

Which implies,

$$f(\tau)g(\tau)h(\tau)+f(\rho)g(\rho)h(\rho)+f(\tau)g(\tau)h(\rho)+f(\rho)g(\rho)h(\tau)\ge f(\tau)g(\rho)h(\tau)+f(\tau)g(\rho)h(\rho))+f(\rho)g(\tau)h(\tau)+f(\rho)g(\tau)h(\rho) \quad ....(1)$$

Multiple both sides of equation (1) by $f(c,tu(t),\alpha,\beta,\eta,q_1)$ and taking -integration of the resulting inequality with respect to τ from 0 to t , we get

$$I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)g(t)h(t)\}+h(\rho)\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)g(t)\}+f(\rho)g(\rho)\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)h(t)\}+f(\rho)g(\rho)h(\rho)\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)\}$$

$$\ge g(\rho)\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)h(t)\}+g(\rho)h(\rho)\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)\}+f(\rho)\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)g(t)h(t)\}+f(\rho)h(\rho)\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)g(t)\}. \quad ....(2)$$

Next, multiplying both sides of (2) by $f(\rho,t,u(\rho),\gamma,\delta,\xi,q_2)$ taking integration of the resulting inequality with respect to ρ from 0 to t, and using definition. This completes the proof of theorem.

$$I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)g(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)g(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)h(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)h(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)g(t)h(t)\}$$

$$\ge I_{q_2}^{\gamma,\delta,\xi}\{u(t)g(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)h(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)g(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)f(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)g(t)h(t)\}+I_{q_2}^{\gamma,\delta,\xi}\{u(t)g(t)\}\,I_{q_1}^{\alpha,\beta,\eta}\{u(t)f(t)h(t)\}.$$

3. SPECIAL CASES AND CONCLUDING REMARKS

In this section, we consider some consequences of the main results derived in the preceding sections. In this section, we consider some consequences of the main results derived in the preceding sections.

Here we derive certain new integral inequalities by setting β = -α and $\delta=-\gamma$, we obtain four integral inequalities involving -Riemann-Liouville fractional integral operators.

$I_{q_2}^{\alpha}\{u(t)f(t)\,g(t)\,h(t)\}\ I_{q_1}^{\alpha}\{u(t)\} + I_{q_2}^{\alpha}\{u(t)f(t)\,g(t)\}\,I_{q_1}^{\alpha,}\{u(t)\,h(t)\} + I_{q_2}^{\alpha}\{u(t)\,h(t)\}\,I_{q_1}^{\alpha,}\{u(t)f(t)h(t)\} + I_{q_2}^{\alpha}\{u(t)\}\,I_{q_1}^{\alpha,}\{u(t)f(t)\,g(t)\,h(t)\}$

$\geq I_{q_2}^{\alpha}\{u(t)\,g(t)\,h(t)\}\,I_{q_1}^{\alpha,}\{u(t)f(t)\} + I_{q_2}^{\alpha}\{u(t)\,f(t)\,h(t)\}\,I_{q_1}^{\alpha,}\{u(t)g(t)\} + I_{q_2}^{\alpha}\{u(t)\,f(t)\}\,I_{q_1}^{\alpha,}\{u(t)g(t)h(t)\} + I_{q_2}^{\alpha}\{u(t)\,g(t)\}\,I_{q_1}^{\alpha,}\{u(t)f(t)h(t)\}$.

REFERENCES

[1] Ahmadmir, N. and Ullah, R.: Some inequalities of Ostrowski and Gruss type for triple integrals on time scales, Tamkang J. Math., 42(4) (2011), 415-426.

[2] Cerone, P. and Dragomir, S.S.: New upper and lower bounds for the Chebyshev functional, J. Inequal. Pure App. Math., 3 (2002), Article 77.

[3] Chebyshev, P.L.: Sur les expressions approximatives des integrales definies par les autres prises entre les mêmes limites, Proc. Math. Soc. Charkov, 2(1882), 93-98.

[4] Dragomir, S.S.: Operator Inequalities of the Jensen,Čebyšev and Grüss Type, Springer Briefs in Mathematics, Springer, New York, 2012.

[5] Dragomir, S.S. and Wang, S.: An inequality of Ostrowski-Grüss' type and its applications to the estimation of error bounds for some special means and for some numerical quadrature rules, Comput. Math. Appl., 13(11) (1997), 15-20.

[6] B. Sroysang, "A study on a new fractional integral inequalities in quantum calculus," *Advanced Studies in theoretical Physics*, vol. 7, no. 14, pp. 689–692, 2013.

[7] Mitrinović, D.S., Pečarić, J.E. and Fink, A.M.: Classical and New Inequalities in Analysis, Kluwer Academic, 1993.

[8] R. P. Agrawal, "Certain fractional q-integrals and q-derivatives," Mathematical Proceedings of the Cambridge Philosophical Society, vol. 66, no.2, pp. 365–370, 1969.

[9] Al. Salam, W. A (1966), Some fractional q- integral and q- derivatives. Proc. Edin. Math. Soc.17, 616-621.

[10] Slater, L.J (1966), Generalized Hypergeometric function, Cambridge University, Press.

[11] Jackson, F. H (1910), On basic double Hypergeometric function, Quart, J. Pure and Appl. Math; 41, 193-203.